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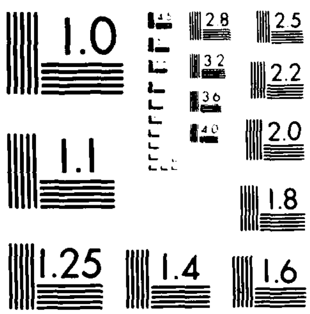

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FOREIGN TECHNOLOGY DIVISION

THE QUESTION OF OPTIMIZATION OF THREE-DIMENSIONAL MANEUVER
OF ORBITAL APPARATUS IN THE ATMOSPHERE

by

V. T. Pashintsev

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

GRAPHICS DISCLAIMER

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THE QUESTION OF OPTIMIZATION OF THREE-DIMENSIONAL MANEUVER OF ORBITAL APPARATUS IN THE ATMOSPHERE.

V. T. Pashintsev.

Page 2.

SUMMARY.

Is examined the problem of the aerodynamic rotation of the plane of the motion of the flight vehicle, which realizes deorbit of ISZ [ИСЗ - artificial earth satellite] into the dense layers of the atmosphere, with its subsequent escape to assigned flight altitude above certain fixed/recorded by point earth's surface. Are indicated the sufficiently simple methods of the construction of the approximate program of control by roll attitude, that makes it possible to satisfy limitations to finite values of phase coordinates.

Introduction.

The study of a question about the optimal control of the orbital apparatus, which accomplishes three-dimensional/space maneuver in the atmosphere, is of interest in connection with the fact that the use of lift-drag ratio on the apparatuses of this type makes it possible to considerably widen the possibilities of their use/application, also, in a number of cases to substantially decrease the general/common/total power expenditures, required for the rotation of the plane of the motion of apparatus, in particular on the rotation of the plane of initial orbit [1].

The efficiency of the use of aerodynamic forces for the accomplishment of the maneuver of orbital apparatus, until now, was studied, as a rule, in connection with the problem about the rotation of the orbital plane. Power expenditures, required for the accomplishment of maneuver, were evaluated according to the value of ideal velocity, necessary for the compensation for aerodynamic drag. It was established/installed, that the aerodynamic method of the rotation of the orbital plane proves to be energetically more advantageous in comparison with rocket-dynamics at the angles of rotation, which exceed $\Delta i=7°-9°$, and as the approximate law of control of attitude of roll γ with a sufficient accuracy can be accepted program γ=const.

In the present work the aerodynamic maneuver of orbital apparatus (Fig. 1) in the initial phase of trajectory of deorbit, which is characterized by the presence of intense reflections from the dense layers of the atmosphere, is investigated. In this case the use of engine thrust is excluded.

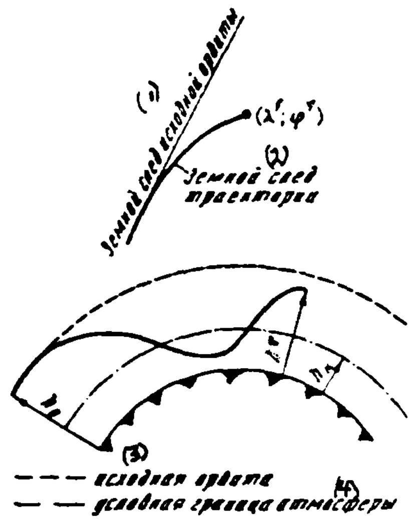


Fig. 1. Profile of maneuver.

Key: (1). Terrestrial track of initial orbit. (2). Terrestrial track of trajectory. (3). initial orbit. (4). conditional boundary of the atmosphere.

Page 4.

FORMULATION OF THE PROBLEM.

In spherical-high-speed coordinate system without taking into account the rotation of the Earth the equations of motion of apparatus, which possesses lift-drag ratio, take the following form:

$$\left.\begin{aligned} \frac{dV}{dt} &= -g_0 \frac{\sigma_x}{2} \rho V^2 - g \sin\theta; \\ \frac{d\theta}{dt} &= \frac{1}{V}\left[g_0 \frac{\sigma_y}{2} \rho V^2 \cos\gamma - \left(g - \frac{V^2}{r}\right)\cos\theta\right], \\ \frac{dh}{dt} &= V \sin\theta; \end{aligned}\right\} \quad (1a)$$

$$\left.\begin{aligned} \frac{d\eta}{dt} &= \frac{1}{V\cos\theta}\left[\frac{g_0 \sigma_y}{2} \rho V^2 \sin\gamma - \frac{V^2 \cos^2\theta}{r} \cos\eta \operatorname{tg}\varphi\right]; \\ \frac{d\varphi}{dt} &= \frac{V\cos\theta \sin\eta}{r}; \\ \frac{d\lambda}{dt} &= \frac{V\cos\theta\cos\eta}{r\cos\varphi}; \end{aligned}\right\} \quad (1b)$$

where

$$\sigma_x = \frac{c_x S}{G}, \quad \sigma_y = \frac{c_y S}{G}, \quad \rho = \rho_0 e^{-zh},$$

$$r = R + h, \quad g = g_0 \frac{R^2}{r^2}, \quad R = 6371{,}21 \cdot 10^3 \text{ м},$$

$$g_0 = 9{,}81 \text{ м/сек}^2, \quad z = \frac{1}{7000} \frac{1}{м};$$

η - heading/course angle between the projection of velocity vector on the plane of the local horizon and the parallel,

φ, λ - geographic latitude and longitude/length,

γ - attitude of roll of the apparatus (positive value γ contributes to an increase of η).

Let us note that the motion of apparatus in the plane of scanning/sweep, described by subsystem (1a), does not depend on the angle of bank.

Let us assume to the piecewise-continuous control $\gamma(t)$ the limitation of the form

$$\gamma_{min} \leqslant \gamma \leqslant \gamma_{max}, \quad (2)$$

is superimposed, and the functional, maximized on the final diversity, isolated in the phase space by equations

$$\left.\begin{aligned} \varphi(t_1) - \varphi^1 = 0; \\ \lambda(t_1) - \lambda^1 = 0; \\ h(t_1) - h^1 = 0, \end{aligned}\right\} \quad (3)$$

is certain value

$$I(\gamma) = \varphi(V(t_1),\ \theta(t_1),\ \eta(t_1)). \quad (4)$$

The control $\gamma(t)$, which maximizes $I(\gamma)$ with the the free t_1, and also the free moment/torque t_0 of the descent of apparatus from the orbit, we will call optimum, and the corresponding phase trajectory x(t), which satisfies conditions (3),—extremal.

Page 5.

APPROXIMATE PROGRAM OF CONTROL OF ROLL ATTITUDE.

We will use conventional assumptions [2], connected with the assumption of the smallness of angle $\theta(t)$ and the smallness of flight altitude h(t) in comparison with a radius R Earth:

$$\sin\theta \sim \theta; \quad \cos\theta \sim 1; \quad r \sim R. \quad (5)$$

Passing to the new independent variable/alternating/variable angular flying range s, with the help of relationship/ratio ds=(V/R)dt let us convert system of equations of motion (1) taking into account (5) to the following form:

$$\left.\begin{aligned} \dot{V} &= -a_x \rho V, \\ \dot{\theta} &= a_y \rho \cos\gamma - \left(\frac{gR}{V^2} - 1\right); \\ \dot{h} &= R\theta; \end{aligned}\right\} \quad (6a)$$

$$\left.\begin{aligned} \dot{\eta} &= a_y \rho \sin\gamma - \cos\eta \,\mathrm{tg}\,\varphi; \\ \dot{\varphi} &= \sin\eta; \\ \dot{\lambda} &= \frac{\cos\eta}{\cos\varphi}; \\ \dot{t} &= \frac{R}{V}, \end{aligned}\right\} \quad (6b)$$

where

$$a_x = g_0 \frac{\sigma_x R}{2}; \quad a_y = g_0 \frac{\sigma_y R}{2}, \quad \left(\frac{a_y}{a_x} = k\right).$$

Entering in accordance with the principle of L. S. Pontriagin's maximum [3], [4], let us register the function of Hamilton H and the conjugated/combined system of equations:

$$H(p_i, x_i, \gamma) = (p_\theta \cos\gamma + p_\eta \sin\gamma)\, a_y \rho - p_V a_x \rho V + p_h R\theta + \\ + \left[-p_\eta \cos\eta \,\mathrm{tg}\,\varphi - p_\theta \left(\frac{gR}{V^2} - 1\right) + p_\lambda \frac{\cos\eta}{\cos\varphi} + p_\varphi \sin\eta\right]; \quad (7)$$

$$\left.\begin{aligned}
&\dot p_V = p_V a_x \rho - p_\theta \frac{2gR}{V^3},\\
&\dot p_\theta = -p_h R,\\
&\dot p_h = -p_V a_x V z \rho + (p_\theta \cos\gamma + p_\eta \sin\gamma)\, a_y \rho z,\\
&\dot p_\eta = -p_\eta \sin\eta\, \mathrm{tg}\,\varphi + p_\lambda \frac{\sin\eta}{\cos\varphi} - p_\varphi \cos\eta,\\
&\dot p_\varphi = p_\eta \cos\eta \frac{1}{\cos^2\varphi} - p_\lambda \frac{\cos\eta \sin\varphi}{\cos^2\varphi},\\
&\dot p_\lambda = 0.
\end{aligned}\right\} \quad (8)$$

Optimal control γ on open set of the allowed values is determined from conditions [3]:

$$\left.\begin{aligned}
&\sin\gamma_{opt} = \frac{-p_\eta}{\sqrt{p_\eta^2 + p_\theta^2}}; \quad \cos\gamma_{opt} = \frac{-p_\theta}{\sqrt{p_\theta^2 + p_\eta^2}};\\
&p_\theta \cos\gamma_{opt} + p_\eta \sin\gamma_{opt} < 0,
\end{aligned}\right\} \quad (9)$$

where $\sqrt{p_\eta^2 + p_\theta^2} > 0$.

Page 6.

Taking into account that in the problem $H(p_i, x_i, \gamma) = 0$, R=const in question, from (7) and (8) it is not difficult to obtain the relationships/ratios following equivalent between themselves:

$$\ddot p_\theta + R \dot p_h = 0, \quad (10)$$

$$\ddot p_\theta + Rz\theta \dot p_\theta + Rz\left(\frac{gR}{V^2} - 1\right) p_\theta = Rzq, \quad (11)$$

where

$$\dot p_h = -p_h R z \theta + p_\theta z \left(\frac{gR}{V^2} - 1\right) - zq, \quad (12)$$

$$q = -p_\eta \cos\eta\, \mathrm{tg}\,\varphi + p_\lambda \frac{\cos\eta}{\cos\varphi} + p_\varphi \sin\eta, \quad Rz = \mathrm{const}. \quad (13)$$

Equation (11) in the range of the velocities, which do not exceed circular ($V<\sqrt{gR}$), describes the process of forced oscillations.

In connection with this function $\ddot{p}_{\theta}(s)$, and consequently [according to (10)], and functior $\dot{p}_h(s)$ they oscillate about the zero values. Therefore, using certain averaged value of function $p_{\theta}(s)$, it is possible to obtain the approximate program of control of roll attitude.

Actually/really, if, for example, value p_{θ} is determined from condition (12), assuming/setting in the first approximation, $p_h(s)$ constant,

$$p_{\theta\, cp} \approx \frac{q + p_{h\, cp} R \theta}{\frac{gR}{V^2} - 1}, \quad p_{h\, cp} \approx \text{const}, \tag{14}$$

then formula for the approximate program of control of roll attitude will take the following form:

$$\operatorname{tg}\gamma \approx \left(\frac{gR}{V^2} - 1\right) \frac{p_{\gamma}}{q + p_{h\, cp} Rz\theta}. \tag{15}$$

Comparison with the formula

$$\operatorname{tg}\gamma = \left(\frac{gR}{V^2} - 1\right) \frac{p_{\gamma}}{q}, \tag{16}$$

of that being determining the optimum program of roll attitude in the mode/conditions of quasi-stationary gliding/planning ($\dot{\theta}=0$) shows that the presence in denominator of expression (15) of value $\rho_{h\,cp}Rz^{b}$ gives further information about a change in the instantaneous value of the flight path angle $\theta(s)$. This, in turn, gives the further possibility to actively affect the flight trajectory due to the appropriate assignment in (15) of the value of free parameter $\rho_{h\,cp}$.

It is characteristic that, in contrast to (15), formula (16) defines attitude of roll γ at those points in the trajectory, in which the second derivative of value p_θ, considered as the function of argument $x=\ln V/V_0$, takes zero values.

Page 7.

Actually/really, function $p_\theta(x)$ with $V<\sqrt{gR}$ satisfies with respect to the variational equation of the form

$$p_\theta'' + \frac{Rz}{a_x^2\rho^2}\left(\frac{gR}{V^2}-1\right)p_\theta = q\,\frac{Rz}{a_x^2\rho^2}, \qquad (17)$$

and, therefore, at points in the trajectory, at which $p_\theta''=0$, value p_θ is determined by the relationship/ratio

$$p_\theta = \frac{q}{\dfrac{gR}{V^2}-1}.$$

Use in formula (15) of the first integrals of system (6) obtained by V. F. Illarionov (8) in the form

$$\left.\begin{aligned} p_{\varphi} &= \frac{p_{\eta}^{j} - p_{\lambda}^{j}\sin\varphi^{j}}{\cos\varphi^{j}}\sin(\lambda-\lambda^{j}) + p_{\varphi}^{j}\cos(\lambda-\lambda^{j}),\\ p_{\eta} &= \frac{p_{\eta}^{j} - p_{\lambda}^{j}\sin\varphi^{j}}{\cos\varphi^{j}}\cos(\lambda-\lambda^{j}) - p_{\varphi}^{j}\sin(\lambda-\lambda^{j})\cos\varphi + p_{\lambda}^{j}\sin\varphi \end{aligned}\right\} \quad (18)$$

makes it possible to express the instantaneous value of the approximate roll attitude in the function of phase coordinates and some constants of integration p_{η}^{j}, p_{φ}^{j}, p_{λ}^{j}, $p_{h\,cp}$, determined from the boundary conditions of problem at the j-end/lead of the trajectory. The need for the integration of interconnected circuit in this case completely is excluded.

One should say that during the determination of the approximate law of control of attitude of roll according to formula (15), valid for open set of the allowed values γ, limitation (2) can be performed via usual "truncation" of value γ.

FURTHER SIMPLIFICATIONS. STRUCTURE OF THE APPROXIMATE CONTROL OF THE MODULUS OF ROLL ATTITUDE IN THE CLASS OF PIECEWISE CONSTANT FUNCTIONS.

In spite of sufficient simplicity of formula (15), the establishment of characteristic properties in the structure of optimal control of roll attitude nevertheless remains difficult. In connection with this let us pass to the examination of the approximate control in the class of the piecewise constant functions.

Let us consider the case of the single insertion/immersion of apparatus in the atmosphere at first. We will count the initial orbit of equatorial. Then latitude φ in (1) will characterize the instantaneous value of the value of parallax relative to initial orbit, the execution of the second of conditions (3) can be provided with the appropriate selection of the moment/torque of the descent of apparatus from orbit t_0.

Will introduce into the examination the conditional boundary of the atmosphere, which corresponds to certain height/altitude

$$h = h_k, \quad (19)$$

higher than which motion can be considered Keplerian. Let us take condition (19) as the sign/criterion, determining end point of time $t = t_k$ on the ascent path,.

Using formulas of Keplerian theory, the not difficult initial problem of reducing to the problem of maximization on the boundary of the atmosphere (at moment/torque $t = t_k$) of the functional of the form

$$I(\gamma) = \varphi(V_k, \theta_k, i_k, h_k) \quad (20)$$

when the limitation of the form is present,

$$\psi(V_k, \theta_k, i_k, h_k) = 0. \quad (21)$$

Page 8.

Thus, let us consider the atmospheric trajectory phase, in which we will assume/set the effect of aerodynamic forces predominating over all remaining forces, which function on the apparatus, and we will consider that the angles η and φ of three-dimensional/space turn in the limits of one immersion into the atmosphere are small:

$$\sin\eta\sim\eta;\quad \cos\eta\sim 1;\quad \sin\varphi\sim\varphi;\quad \cos\varphi\sim 1. \tag{22}$$

Under these conditions the system of equations (6) takes the following form:

$$\left.\begin{aligned} \dot V &= -a_x \rho V, \\ \dot\theta &= a_y \rho \cos\gamma, \\ \dot h &= R\theta; \end{aligned}\right\} \tag{23a}$$

$$\left.\begin{aligned} \dot\eta &= a_y \rho \sin\gamma, \\ \dot\varphi &= \eta, \\ \dot\lambda &= 1. \end{aligned}\right\} \tag{23b}$$

We will assume that the scalar function $\varphi(V(t_1), \theta(t_1), \eta(t_1))$ in (4), determined at end point of time t_1, is such, that the maximization of the final velocity $V(t_1)$ in the constant/invariable ones $\theta(t_1)$ and $\eta(t_1)$ continuously corresponds to the maximization of functional $I(\gamma)$. In that case as this is not difficult to show, the solution of the initial problem about the maximization of functional (20) under the condition of connection/communication (21) is found among the solution set, which correspond to the problem about the maximization of the velocity at the end of atmospheric section V_k at height/altitude $h=h_k$ with different fixed values of final parameters θ_k, η_k and φ_k. In connection with this the unknown solution can be found with the method of the parametric optimization of values θ_k, η_k and φ_k.

We will use this fact and let us consider the case of "small" optimum (required) angles $\Delta i_k(\eta_k, \varphi_k)$ of the aerodynamic rotation of the plane of motion, which do not exceed in the modulus/module of the corresponding angles of rotation $\Delta\tilde{i}_k$, obtained in the problem about maximization V_k at height/altitude $h=h_k$ in free parameters η_k and φ_k, constant/invariable parameter θ_k and constant attitude of roll γ.

Thus,

$$|\Delta i_{k\,opt}| \leqslant |\Delta\tilde{i}_k|. \quad (24)$$

In this case reaching/achievement of fixed/recorded parameters η_k and φ_k can be, obviously, provided without the further expenditures of energy due to the appropriate programming of the angle of the bank (exchange of sign γ does not affect motion in the plane of scanning/sweep).

Taking into account of the aforesaid, value η_k and φ_k in (6) it is possible to unfasten and subsequently to examine only system of equations

$$\left.\begin{aligned} \dot{V} &= -a_x \rho V, \\ \dot{\theta} &= a_x \rho k \cos\gamma, \\ \dot{\rho} &= -Rz\theta\rho. \end{aligned}\right\} \quad (25)$$

Let us note that since the trajectory of apparatus, which is characterized by reflection from the dense layers of atmosphere ($\dot{\theta}>0$), interests us, we will assume that the value maximum on the modulus/module of roll attitude does not exceed certain limiting value $|\gamma_{пред}|$:

$$|\gamma|_{max} \leqslant |\gamma_{пред}| < \frac{\pi}{2}. \quad (26)$$

Page 9.

Accepting as the argument the monotonically increasing variable $\theta(s)$, we convert system (25) to the form:

$$\frac{d\ln V}{d\theta}=-\frac{1}{k\cos\gamma},\qquad \frac{d\rho}{d\theta}=-\frac{z\theta}{a_z k\cos\gamma}. \tag{27}$$

As a result we come to the following task: it is necessary to determine the control γ on set of allowed values (26), which maximizes functional $\ln V_k$ at the fixed final values variable/alternating/variable ρ and θ, equal to

$$\rho_k=\rho^*,\quad \theta_k=\theta^*.$$

Initial parameters V_e, θ_e and ρ_e are assigned.

According to the principle of L. S. Pontriagin's maximum, the function of Hamilton and the conjugated/combined system of equations take the following form:

$$H=-\left(p_1+p_2\frac{z\theta}{a_z}\right)\frac{1}{k\cos\gamma}; \tag{28}$$

$$\dot p_1=0;\quad \dot p_2=0. \tag{29}$$

Constant in (29) value p_1, which corresponds to coordinate $\ln V$.

let us assign as follows:

$$p_1 = \text{const} \leqslant 0. \tag{30}$$

Then optimality condition of control γ, which consists of the minimization of Hamilton's function $H(\gamma)$, takes the form:

$$|\gamma_{opt}| = \begin{cases} |\gamma|_{max} & \text{при } \vartheta(\theta) < 0, \\ |\gamma|_{min} & \text{при } \vartheta(\theta) > 0, \end{cases} \tag{31}$$

Key: (1). with.

where

$$\vartheta(\theta) = -\left(p_1 + p_2 \frac{z\theta}{a_x}\right) \tag{32}$$

there is a function of the changeover of control γ.

To the moment/torque of the changeover of boundary $|\gamma|$ corresponds certain value of argument, determined from the condition of equality to zero functions of changeover of the formula

$$\theta_* = -\frac{p_1}{p_2}\frac{a_x}{z}. \tag{33}$$

From formula (33) taking into account (30) it follows that

$$\theta_* \begin{cases} \leqslant 0 & \text{при } p_2 < 0, \\ \geqslant 0 & \text{при } p_2 > 0. \end{cases} \tag{34}$$

Key: (1). with.

Page 10.

If we now take into account that to the condition of the positiveness of the function of changeover it corresponds:

$$\theta(s)\begin{cases}\geqslant 0 \text{ при } p_2<0,\\ \leqslant 0 \text{ при } p_2>0,\end{cases} \quad (35)$$

Key: (1). with.

that of (30), (34) and (35) easily is established that independent of $|p_2|$ the minimum on the modulus/module roll attitude is optimum on the ascent path $[\theta(s)\geqslant 0]$ when $p_2<0$. But if $p_2>0$, then attitude of roll $|\gamma|=|\gamma|_{min}$ is optimum respectively on the descending branch of trajectory $[\theta(s)\leqslant 0]$.

We will consider that $|\gamma|_{min}=0$. Then taking into account the uniqueness of the moment/torque of the changeover of limiting angle of bank from the aforesaid easily is established/installed the following structure of optimal in (25) control of the modulus/module of attitude of roll (Fig. 2):

$$p_2<0, \quad |\gamma_{opt}|=\begin{cases}|\gamma|_{max} & \text{при } \theta_e\leqslant\theta(s)\leqslant\theta_*; \quad \theta_*\leqslant 0,\\ 0 & \text{при } \theta_*\leqslant\theta(s)\leqslant\theta_k;\end{cases} \quad (36)$$

$$p_2>0, \quad |\gamma_{opt}|=\begin{cases}0 & \text{при } \theta_e\leqslant\theta(s)\leqslant\theta_*; \quad \theta_*\geqslant 0,\\ |\gamma|_{max} & \text{при } \theta_*\leqslant\theta(s)\leqslant\theta_k.\end{cases} \quad (37)$$

Key: (1). with.

In each specific case the optimality of one or the other type of control is determined by the boundary conditions of task.

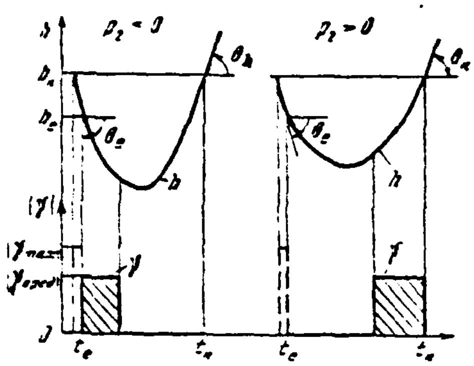
It is possible to show that the physical sense of parameter p_2 the task in question is in connection with the following:

$$p_2 = -\frac{\partial I_{max}(\rho_k)}{\partial \rho_k}; \quad I \equiv \ln \frac{V_k}{V_e}. \tag{38}$$

In view of the linearity of system of equations (27) function $I_{max}(\rho_k)$ is convex. Hence, for example, it follows that in the region of the solvability of task (Fig. 3), determined (at the fixed value of angle θ_k) by the inequality

$$\rho_{k\,min} \leqslant \rho_k \leqslant \rho_{k\,max}, \tag{39}$$

in segment $[\rho_{k\,min}; \rho_{k\,max}]$ there is a unique point $\rho_k = \rho_{k*}$, at which functional $\frac{V_k}{V_e}$ on set of permissible controls U reaches absolute maximum $[(I_{max}(\rho_{k*}) = \sup I(\gamma)]$.



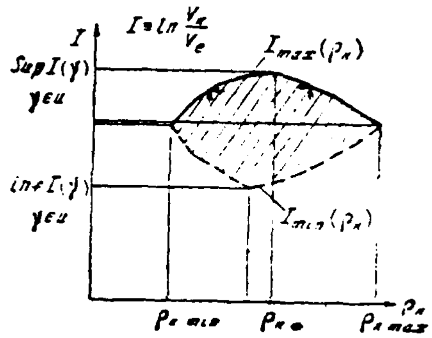


Fig. 2.

Fig. 3.

Fig. 2. Structure of optimal roll control during single insertion/immersion of apparatus in the atmosphere.

Fig. 3. Region of solvability of task.

Page 11.

Therefore

$$\gamma \in U$$

$$\frac{\partial I_{max}(\rho_k)}{\partial \rho_k} \begin{cases} >0 & \text{при}^{(1)}\ \rho_{k\,min} \leqslant \rho_k \leqslant \rho_{k*}, \\ <0 & \text{при}\ \rho_{k*} \leqslant \rho_k \leqslant \rho_{k\,max}. \end{cases} \qquad (40)$$

Key: (1). with.

Condition (40) taking into account (38) determines the dependence of the sign of value p_2 and, consequently, also the structure of optimal control γ from the boundary conditions of task.

For determining the limiting values of final density $\rho_{k\,max}$ and $\rho_{k\,min}$ at fixed value θ_k, it is necessary respectively to maximize and to minimize in (27) coordinate ρ_k at the free final flight speed. With the help of the principle of L. S. Pontriagin's maximum we obtain the appropriate programs of the control:

$$\rho_k=\rho_{k\,min}, \qquad |\gamma_{opt}|=\begin{cases}|\gamma_{пред}| & \text{при (1) } \theta(s)\leqslant 0,\\ 0 & \text{при } \theta(s)\geqslant 0;\end{cases} \qquad (41)$$

$$\rho_k=\rho_{k\,max}, \qquad |\gamma_{opt}|=\begin{cases}0 & \text{при (1) } \theta(s)\leqslant 0,\\ |\gamma_{пред}| & \text{при } \theta(s)\geqslant 0.\end{cases} \qquad (42)$$

Key: (1). with.

Value ρ_{k*} corresponds to the value of final density ρ_k, obtained as a result of the solution of the problem about the maximum of value $\ln V_k$ in free parameter ρ_k and fixed/recorded θ_k. In this case we have $p_2=0$, and therefore when $\rho_k=\rho_{k*}$

$$|\gamma_{opt}|\equiv 0. \qquad (43)$$

Integrating system (27) under the laws of control (41)-(43), taking into account (38) and (40) we come to the following inequalities, which isolate the regions of the boundary conditions of tasks, which correspond to sign-constancy of parameter p_2 (Fig. 4):

$$p_2<0, \qquad \left(\theta_k^2-\frac{\theta_e^2}{\cos\gamma_{\text{пред}}}\right)\frac{1}{ak}\leqslant\rho_k-\rho_e\leqslant\frac{\theta_k^2-\theta_e^2}{ak}; \qquad (44)$$

$$p_2>0, \qquad \frac{\theta_k^2-\theta_e^2}{ak}\leqslant\rho_k-\rho_e\leqslant\left(\frac{\theta_k^2}{\cos\gamma_{\text{пред}}}-\theta_e^2\right)\frac{1}{ak}, \qquad (45)$$

where $a=\frac{a_r}{z}$.

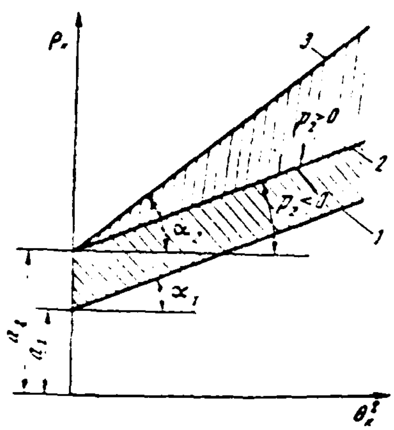


Fig. 4.

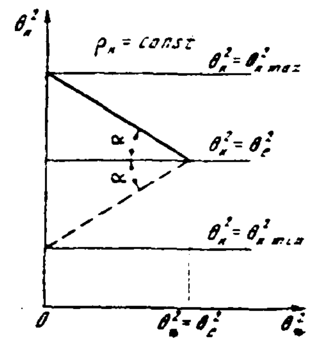


Fig. 5.

Fig. 4. To determination of regions constancy of sign of parameter p_2. 1 - program of the control of form (41), 2 - program of the control of form (43), 3 - program of the control of form (42),

$$a_1=p_e-\frac{\theta_e^2}{ak\cos\gamma_{\text{пред}}};\quad a_2=p_e-\frac{\theta_e^2}{ak};$$

$$\alpha_1=\operatorname{arctg}\frac{1}{ak};\quad \alpha_2=\operatorname{arctg}\frac{1}{ak\cos\gamma_{\text{пред}}}$$

Fig. 5. Dependence $\theta_k^2(\theta_*^2)$ for two types of optimal control --- optimal control with $p_2<0$ form (36), --- the same, with $p_2>0$ form (37),

$$\alpha=\operatorname{arctg}\left(1-\frac{1}{\cos\gamma_{\text{пред}}}\right)$$

Page 12.

By the use of inequalities (44) and (45) is determined the sign of parameter p_2 for any values ρ_k and θ_k, assigned in the region of the solvability of task, and therefore the corresponding type of the optimum program of control of the modulus/module of attitude of roll [(36) or (37)].

It is characteristic that when $\rho_k = \rho_e$ in each of the regions of boundary conditions (44) and (45) are valid the relationships/ratios of form $|\theta_k| \geqslant |\theta_e|$ and $|\theta_k| \leqslant |\theta_e|$ respectively. In connection with this the program of control of the attitude of roll of form (36) (Fig. 5) corresponds to the more intense flight of apparatus from the dense layers of the atmosphere (to high values of final angle $|\theta_k|$).

Let us note that on the basis of the physical sense of parameter p_2, to condition $p_2<0$ corresponds decrease $V_{k\,max}$ with decrease ρ_k when $\theta_k = const$, or, that nevertheless, decrease $V_{k\,max}$ with increase θ_k when $\rho_k = const$. Hence the sense of mutual (converted) task lies in the fact that in fixed/recorded parameters V_k and ρ_k value θ_k with $p_2<0$ ($p_2>0$) should be maximized (to minimize). It is analogous in fixed/recorded parameters V_k and θ_k with $p_2<0$ ($p_2>0$) should be minimized (to maximize) value ρ_k. The latter is shown in Fig. 3 by arrows/pointers.

Fig. 6 depicts the typical dependence of the finite values of the parameters of trajectory, which correspond to flight altitude h^1=100 km, and also apogee altitudes of trajectory h_a from the flight path angle θ_k, determined at the height taken as the conditional boundary of the atmosphere, equal to $h_k = 70$ км. The value of lift-drag ratio is accepted equal to K=1.5, and $\gamma_{max} = 87°$ (sign γ=const).

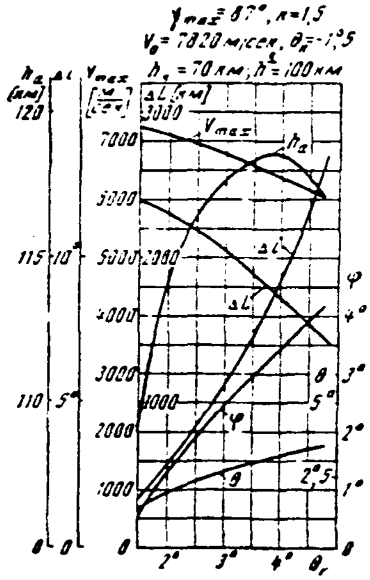


Fig. 6.

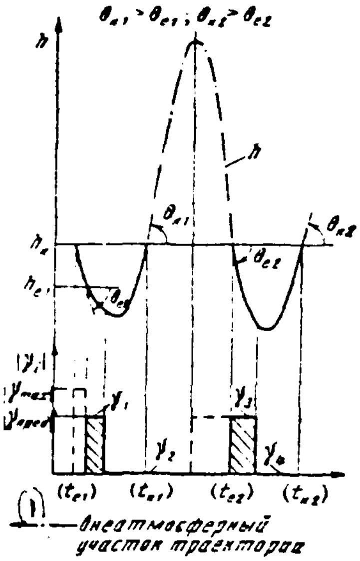


Fig. 7.

Fig. 6. Effect θ_k on parameters of trajectory, which correspond to flight altitude h=h' under law of control of form (36).

Key: (1). m/s.

Fig. 7. Structure of optimal roll control during double insertion/immersion of apparatus in the atmosphere.

Key: (1). Outer-atmosphere trajectory phase.

Page 13.

It is characteristic that as a result of an increase in the duration of flight at the maximum roll attitude with increase/growth θ_k the angle of rotation of the plane of motion Δi, and also the value of lateral deviation φ continuously grow.

Using the results obtained above, let us construct the approximate program of the piecewise constant control of the modulus/module of roll attitude for the case, when

$$|\Delta i_{k\,opt}| > |\Delta \bar{i}_k|. \qquad (46)$$

For this purpose in the examination of the complete system of equations of motion (6) we will use the specific character of the programs of control, close to the optimum ones, in the following two tasks, which are special cases of the task in question:

a) the task about reaching/achievement of fixed/recorded angle θ_k with the maximum final rate in free parameters η_k and φ_k (free angle Δi_k);

b) the task about the rotation of the plane of motion to preset angle $\Delta i_k(\eta_k, \varphi_k)$ with the maximum final rate in free parameter θ_k.

Specifically, taking into account that in the second task as this was shown V. P. Plokhikh, the program of the attitude of roll of form γ=const on the functional is sufficiently close one to the optimum, let us represent the general routine of control in the form (36) and (37), assuming that the minimum value of modulus/module $|\gamma|$ in (2) corresponds to certain value $|\breve{\gamma}|$, necessary for the rotation of velocity vector to angle $\delta\Delta i_k = |\Delta i_{k\,opt}| - |\tilde{\Delta} i_k|$.

Thus, as a result of replacing limitation (2) by the inequality

$$|\breve{\gamma}| \leqslant |\gamma| \leqslant |\gamma|_{max} \tag{47}$$

the approximate program of roll attitude in the class of the piecewise constant functions takes the following form:

$$p_1 < 0, \qquad |\gamma| = \begin{cases} |\gamma|_{max} \ ^{(1)}\text{при } \theta_c \leqslant \theta(s) \leqslant \theta_*; & \theta_* \leqslant 0; \\ |\breve{\gamma}| \ ^{(1)}\text{при } \theta_* \leqslant \theta(s) \leqslant \theta_k; \end{cases} \tag{48}$$

$$p_1 > 0, \qquad |\gamma| = \begin{cases} |\breve{\gamma}| \ ^{(1)}\text{при } \theta_c \leqslant \theta(s) \leqslant \theta_*; & \theta_* \geqslant 0; \\ |\gamma|_{max} \ ^{(1)}\text{при } \theta_* \leqslant \theta(s) \leqslant \theta_k. \end{cases} \tag{49}$$

Key: (1) with.

In connection with task "b" in (48) we have $\theta_* = \theta_c$, and in (49), correspondingly, $\theta_* = \theta_k$.

One should say that since the value of angle $\delta\Delta i_k$ usually is previously unknown, when $\delta\Delta i_k \neq 0$ the parameter $|\breve{\gamma}|$ in (47) is subject to optimization.

Let us note finally that from the linearity of the system of equations of motion (27) the analogy in the structure of control of the modulus/module of roll attitude easily follows in the atmospheric trajectory phases in the case of the double insertion/immersion of apparatus in the atmosphere. In this case the type of the approximate program of control [(36) or (37)], adjusted as a result of the examination of the task about the maximization of the flight speed at the end of each of the atmospheric trajectory phases at the fixed values of the angles of arrival θ_{k1opt} and θ_{k2opt}, depends on the value of the latter. Thus, Fig. 7 shows the structure of control in connection with the case, when $|\Delta i_{k\,opt}|\leqslant|\tilde{\Delta} i_k|$, $|\theta_{k1}|>|\theta_{e1}|$, $|\theta_{k2}|>|\theta_{e2}|$.

In the examination of the real motion, described by a precise system of equations (1), it is convenient to use the fact that the program $\gamma\equiv\tilde{\gamma}$ is dividing two types of the approximate in (23) control, which is characterized either by the amplification of the fluctuations of flight altitude [formula (36)], or by damping the fluctuations [formula (37)indicated].

Page 14.

In this case as the fundamental characteristic of the approximate control γ apogee altitude of the outer-atmosphere ellipse of trajectory $h_a = h_a(\theta_k, V_k, \rho_k)$, can be accepted since when ρ_k=const. (in view of freedom of movement in the plane of scanning/sweep from the angle of bank) parameters θ_k and V_k are wholly determined by the structure of control of the modulus/module of roll attitude. Then, for example, in the case of the single insertion/immersion of apparatus in the atmosphere when $h_{a\,\gamma=\bar{\gamma}} \leqslant h^1$, should be used a structure of the control of form (36), and when $h_{a\,\gamma=\bar{\gamma}} \geqslant h^1$ — respectively form (37). It is characteristic that if in the first case to limitation of the type of equality $h(t_1)=h^1$ to final flight altitude $h(t_1)$ is equivalent limitation of the type of the conditional inequality of form $h(t_1) \geqslant h^1$, then in the second case the limitation of form $h(t_1) \leqslant h^1$ is equivalent to it.

EXAMPLE

Let us consider the task about the rotation of the plane of motion with the yield condition at the end of the maneuver into the point (λ^1, φ^1, h^1), with the maximum final rate.

The initial parameters of motion we will consider the following: $h(0)$=130 km; $V(0)$=7820 m/s; $\theta(0)=-1°,5$; $\gamma(0)=0$.

As the limitation to the roll attitude let us take inequality of $|\gamma| \leq 87°$.

For the construction of the program of control in the class of the piecewise-continuous functions we will use formula (15).

Taking into account $p_{\gamma}(t_1)=0$ with the designations of the form

$$\begin{vmatrix} \cos\xi\sin\beta \\ \cos\xi\cos\beta \\ \sin\xi \end{vmatrix} = \begin{vmatrix} p_{\varphi}(t_1) \\ p_{\lambda}(t_1) \\ p_{h\,кр}R \end{vmatrix} \frac{1}{\sqrt{p_{\varphi}^2(t_1)+p_{\lambda}^2(t_1)+(p_{h\,кр}R)^2}}$$

from relationships/ratios (13), (15) and (18) we obtain

$$\mathrm{tg}\,\gamma = \frac{A\cos\xi\sin\beta + B\cos\xi\cos\beta}{D\cos\xi\sin\beta - E\cos\xi\cos\beta + 5\sin\xi}, \qquad (50)$$

where $A=\cos\varphi\sin(\lambda-\lambda^1)$,
$B=\mathrm{tg}\,\varphi^1\cos\varphi\cos(\lambda-\lambda^1)-\sin\varphi$,
$D=\sin\varphi\cos\eta\sin(\lambda-\lambda^1)+\sin\eta\cos(\lambda-\lambda^1)$,
$E=\cos\varphi\cos\eta+\mathrm{tg}\,\varphi^1[\sin\varphi\cos\eta\cos(\lambda-\lambda^1)-\sin\eta\sin(\lambda-\lambda^1)]$.

As a result the program of roll control proves to be biparametric. The parameters ξ and β are selected from the condition of falling in point (φ^1, λ^1, h^1).

In the case in question is realized the double insertion/immersion of apparatus in the atmosphere (Fig. 8), and change γ leads to the amplification of the fluctuations of flight altitude h, since

$$h_{s1} > h_{\gamma=0}, \quad h_{\gamma=0} < h^1. \tag{51}$$

Let us compare the now obtained program of control with the approximate control γ (t) in the class of the piecewise constant functions. Taking into account (51), let us consider the program of the control of form (48), assuming/setting attitude of roll $\breve{\gamma}$ by identical for each of the sections of the insertion/immersion of apparatus in the atmosphere. As a result we come to the four-stage approximation of function $\gamma(t)$: $|\gamma_1|=|\gamma_3|=|\gamma|_{max}, \gamma_2=\gamma_4=\breve{\gamma}$. The unknown parameters are modulus/module $|\breve{\gamma}|$, and also two moments/torques of changeover $|\gamma|_{max}$ to value $\gamma=\breve{\gamma}$.

Page 15.

From Fig. 8 it is evident that the obtained law γ(t) on the whole correctly reflects the behavior of stepless control. The error in the value of the maximized final rate, obtained as a result of a similar approximation, composes the value of order 1%. It is obvious that the accuracy of approximation can be increased, if we, for example, γ_2 and γ_4 optimize independent of each other.

It should be noted that in spite of a larger number of free parameters, the construction of the piecewise constant program of control indicated proves to be simpler in comparison with the procedure of the use of formula (50).

REFERENCES

1. I. O. Mel'ts. Optimum aerodynamic maneuver for changing the orbital plane at the near-circular speeds. "Engineering journal", AH CCCP, Vol. 14, iss. 1, 1964.

2. V. A. Yaroshevskiy. The approximate computation of the trajectories of entry into the atmosphere. "Space research", 1964, VII-VIII, Vol. 2, No 4.

3. L. S. Pontryagin, V. T. Boltyanskiy, V. V. Gamkrelidze, Ye. F. Mishchenko. Mathematical theory of optimum processes. Fizmatgiz, 1961.

4. L. I. Rozonoer. Principle of L. S. Pontriagin's maximum in the theory of optimum systems. I-III, automation and telemechanics, Vol. XX, No 10-12, 1959.

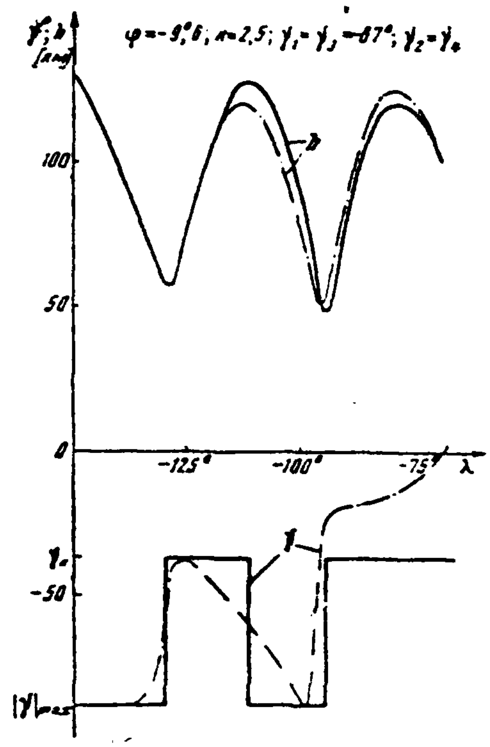


Fig. 8. Optimum stepped approximation of control γ (s).